Generic strong coupling behavior of Cooper pairs in the surface of superfluid nuclei

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With realistic HFB calculations, using the D1S Gogny force, we reveal a generic behavior of concentration of small sized Cooper pairs (2-3 fm) in the surface of superfluid nuclei. This study confirms and extends previous results given in the literature that use more schematic approaches.

The opportunities offered by the new radioactive beam facilities to study the properties of weakly bound nuclei with large neutron skins or halos triggered new interest for the issue of space correlations induced by the formation of Cooper pairs. The spatial correlations of Cooper pairs in superfluid nuclei have not been extensively studied in the past, but nevertheless a certain number of investigations, some rather early, do exist. Mostly this was done for the single Cooper pair problem. For example the rms diameter of the extra neutron pair in ^{18}O is shown as a function of the nuclear radius by Ibarra et al. [1]. One sees a strong minimum in the nuclear surface, indicating an rms separation between the two active neutrons of the order of 2-3 fm. A similar behaviour was found later by Catara et al. [2] and Ferreira et al. [3] for a neutron pair in ^{206}Pb and ^{210}Pb. More recently, there are also many investigations of the single Cooper pair problem in the halo nucleus ^{11}Li [4, 5].

One of the rare papers where spatial correlations of Cooper pairs are investigated in superfluid nuclei is the one of Tischler et al. [6] where the probability distribution of the pairs is shown as a function of the center of mass $R = \frac{1}{2}|\vec{r}_1+\vec{r}_2|$ and the relative distance of the nucleons in the pairs $r = |\vec{r}_1 - \vec{r}_2|$ with $(\vec{r}_1, \vec{r}_2)$ the coordinates of two nucleons. They showed that in the open shell isotope ^{114}Sn one also finds Cooper pairs with short range space correlations, like in one pair systems. They confirmed also the finding of Catara et al, i.e., the important role played by the parity mixing for inducing short range space correlations. Most of those older works were, however, done using rather schematic models and/or pairing forces. There exists, however, one study with a realistic pairing force (i.e., the Gogny interaction) by Barranco et al. [7], dedicated to nuclei embedded in a neutron gas, a system found in the inner crust of neutron stars. One of the first systematic analyses of strong di-neutron spatial correlations induced by the pairing interaction was done recently by Matsuo et al.[8], using a zero range pairing force. The study of nuclear surface pairing properties was also the aim of several half infinite matter investigations [9], [10]. It was found that the pair density reaches out further than the ordinary density but neither the local coherence length nor the probability distribution of the pairs were calculated.

The aim of the present work is to verify how much all these relative scattered pieces of information withstand a general study of superfluid nuclei using one of the most performant HFB approaches, that is employing the finite range Gogny D1S-interaction [12]. As a matter of fact we will see that many of the earlier findings are qualitatively or even quantitatively confirmed. Indeed, we will show that the strong concentration of pair probability of small Cooper pairs in the nuclear surface is a quite general and generic feature and that nuclear pairing is much closer to the strong coupling regime [8, 13] than previously assumed.

We will start by explaining shortly how the spatial properties of nuclear pairing are investigated within the HFB approach and then we shall present our results and conclusions. For further understanding of the phenomena, a simple semiclassical analytic model for nuclear pairing will be also considered.

It is well known [11] that pairing correlations can be adequately studied with the Cooper-pair probability $|\kappa|^2$, where (in standard notation):

$$\kappa_{\sigma,\sigma'}(\vec{r}_1,\vec{r}_2) = < HFB|\psi_{\sigma'}(\vec{r}_2)\psi_{\sigma}(\vec{r}_1)|HFB > \quad , \qquad (1)$$

is the anomalous density matrix or pairing tensor in r-space, calculated with the HFB-wave function. Following Refs. [2, 6], we shall also consider the quantity:

$$P(R,r) = R^2 r^2 |\kappa(R,r)|^2, \qquad (2)$$

which is the pair probability averaged over the angle between $\vec{R}$ and $\vec{r}$ and multiplied by the phase space factors $R^2 r^2$. This quantity is important since it determines the two-particle spectroscopic factor [14] and other expectation values of two-body operators (e.g., pairing energy). Let us mention that Eq.(2) is formally the same for the single Cooper pair problem [1, 2, 3, 5] and for the case of Cooper pairs in a condensate [6].

In what follows, we shall consider the HFB expression of the pairing tensor κ in center of mass and relative coordinates given by:

$$\begin{aligned}\kappa(\vec{R},\vec{r}) &= \frac{1}{4\pi} \sum_{n_1,n_2,l_1 j_1} \kappa^{l_1 j_1}_{n_2,n_1} \\ &\times \sum_{nNl} (-)^l (\frac{2l+1}{2l_1})^{1/2} u_{nl}(r/\sqrt{2}) u_{Nl}(\sqrt{2}R) \\ &\times P_l(cos\hat{rR}) < nlNl;0|n_1 l_1 n_2 l_1;0 > \quad ,\end{aligned} \qquad (3)$$

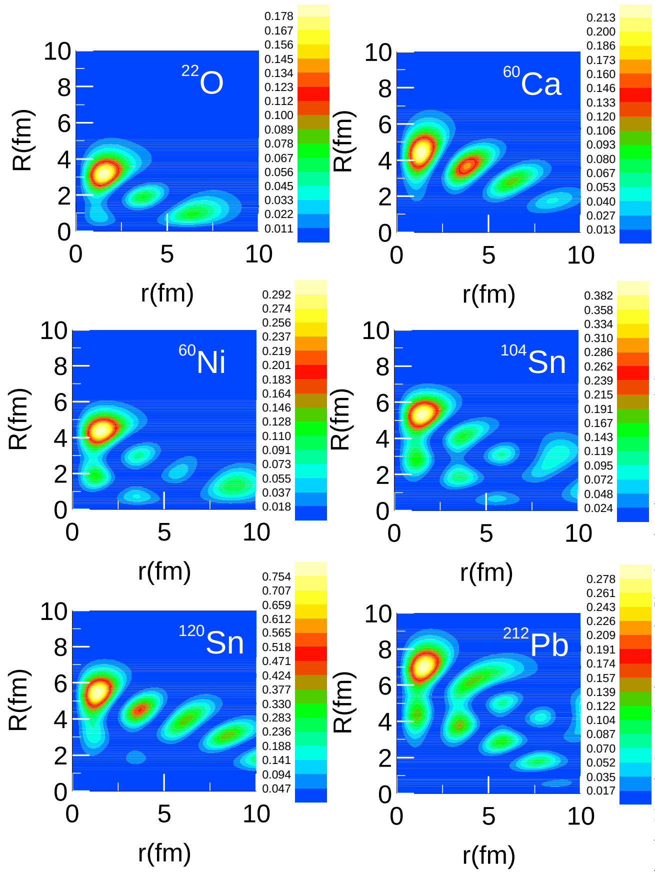

FIG. 1: $P(R,r)$ calculated with HFB-D1S for ^{22}O,^{60}Ca, ^{60}Ni, ^{104}Sn, ^{120}Sn, ^{212}Pb. Scales have been multiplied by a factor of 10^2.

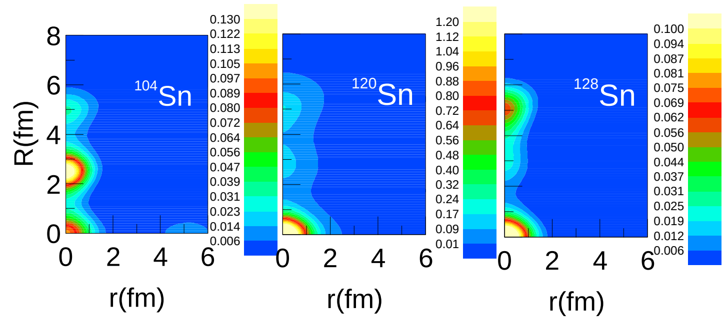

FIG. 2: $|\kappa(R,r)|^2$ calculated with HFB-D1S for ^{104}Sn, ^{120}Sn, ^{128}Sn. Scale has been multiplied by a factor of 10^6.

where $< nlNl; 0|n_1l_1n_2l_1; 0 >$ is the Brody-Moshinski bracket, $u_{nl}(r)$ are the radial wave functions of the harmonic oscillator and $\kappa^{lj}_{n'n}$ is the matrix of the pairing tensor for a given angular momentum lj. As defined here, the latter has an intrinsic parity $(-)^l$. The HFB calculations are performed with the D1S Gogny force [15]. In the calculations a basis with 15 harmonic oscillator shells have been considered. The contour-lines of $P(R,r)$ for various superfluid nuclei are shown in FIG.1. The striking feature is that for all these nuclei the same scenario, with only slight modulations, emerges: the probability $P(R,r)$ is strongly concentrated in the surface with a small diameter of the pairs of the order of $2-3fm$. In FIG.1, we show nuclei close to the neutron drip-line (^{60}Ca) as well as nuclei closer to stability. Seemingly, there is no essential difference in the behavior of $P(R,r)$ between very neutron-rich nuclei and stable ones. This fact explains why one finds in all the superfluid nuclei a high probability for two-neutron transfer reactions.

To conclude from the strong concentration of $P(R,r)$ in the surface that Cooper pairs are mostly sitting in the surface of the nucleus may be, however, a bit misleading. In FIG.2, we show the pair probability $|\kappa(R,r)|^2)$ without the factor R^2r^2 which enters in the probability $P(R,r)$. One can notice that for all tin isotopes the Cooper pairs have very small extension in r-direction throughout the nuclear radius. However, the distribution in R is rather different in the three isotopes. The difference comes from the localisation properties of the single-particle shells which are closest to the chemical potential. Thus, the pronounced concentration of $|\kappa(R,r)|^2$ around R=5 fm in ^{128}Sn is due to the surface localisation of the single-particle wave function $1h_{11/2}$, which becomes much closer to the chemical potential in this isotope compared to lighter ones. One can also notice that in $^{120,128}Sn$ the pair probability has a sizeable value for small values of R, which comes mainly from the contribution of the state $3s_{1/2}$ to pairing correlations. If we have had chosen the neutron deficient Pb-isotopes, in which there is no s-state in the major shell, one would rather see a depression of pair probability at the origin. Therefore, to say where the Cooper pairs are preferentially located in nuclei is a somewhat subtle question because the answer depends rather strongly on the shell structure (see also [16]). The shell structure dependence of $|\kappa(R,r)|^2$ is largely washed out by the phase factor r^2R^2 when $P(R,r)$ is calculated. We shall make further investigations on this issue in a future work. However, we want to point out again that in all expectations values, like e.g. the pairing energy and spectroscopic factors, it is $P(R,r)$ which counts and not $|\kappa(R,r)|^2$.

The fact that the Cooper pairs with small size are concentrated in the surface can be also seen from the dependence of the coherence length on the center of mass of the pairs. The coherence length is defined as:

$$\xi(R) = \frac{(\int r^4|\kappa(R,r)|^2dr)^{1/2}}{(\int r^2|\kappa(R,r)|^2dr)^{1/2}} \qquad (4)$$

It is shown for various nuclei in FIG.3. One sees well defined and pronounced minima at $\xi \sim\ 2-3fm$ for R of the order of the surface radius. As we have already mentioned, a small coherence length in the case of a single Cooper pair has already been found previously for ^{18}O in Ref.[1]. It is also the case for the Cooper pair in ^{11}Li [4, 5]. Our calculations did not allow to go much beyond the minima because of the employed harmonic oscillator basis what becomes inaccurate far outside the nuclear radius. However, the position of the minima is always clearly identified and seen to be

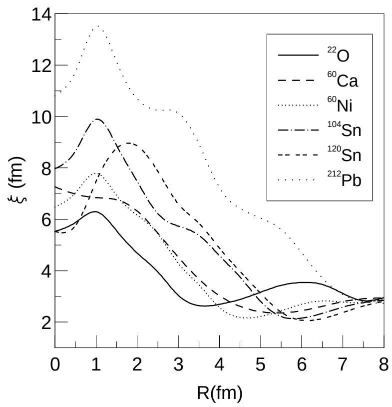

FIG. 3: Coherence length $\xi(R)$ for ^{22}O,^{60}Ca, ^{60}Ni, ^{104}Sn, ^{120}Sn, ^{212}Pb.

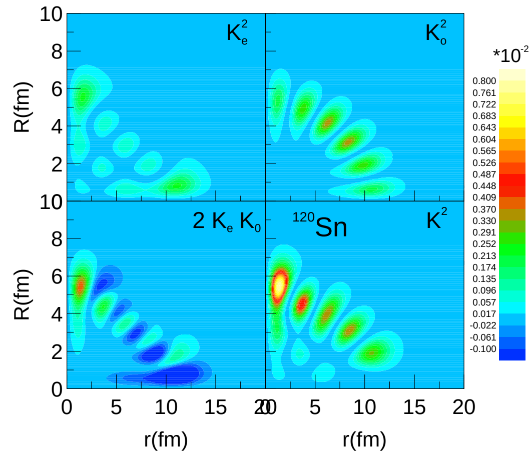

FIG. 4: Contributions of definite parity to $P(R,r)$ calculated with HFB-D1S for ^{120}Sn.

similar in all cases. What is surprising is that the size of the Cooper pairs starts to decrease already well inside, around $R = 2fm$. Moreover, the decrease towards the surface is approximately linear.

In order to demonstrate that the strong concentration of small Cooper pairs in the surface of the nuclei is not a trivial effect, we decompose $\kappa(R,r)$ in a part $\kappa_e(R,r)$ which contains only even parity wave functions and a part $\kappa_o(R,r)$ which contains only the odd parity ones, i.e., $\kappa(R,r) = \kappa_e(R,r)+\kappa_o(R,r)$. In FIG.4, we show what are the probability distributions for $P_e(R,r)$, $P_o(R,r)$ and $P_{eo}(R,r)$ in the case of ^{120}Sn. The quantity $P_{eo}(R,r)$ corresponds to the interference term $2\kappa_e\kappa_o$. From FIG.4 one can see that selecting only either even or odd parity states in $\kappa(R,r)$ has a strong delocalisation effect on the Cooper pairs: they are democratically distributed with respect to an interchange of R and r variables (one should notice that in Eq.(3) the symmetry between R and r involves a factor 2, which comes through the standard definition of Brody-Moshinsky transformation). So no small Cooper pairs in the nuclear surface are prefered at all in those cases. The concentration only shows up, when even and odd parity states are mixed. This is clearly revealed in looking at the interference term $P_{eo}(R,r)$. We see that it is negative for regions close to the r-axis and positive close to the R-axis. We checked that this scenario stays the same for all other superfluid nuclei considered. This scenario was also nicely described in the papers by Catara et al. [2] and Tischler et al. [6]. Mixing of parities naturally occurs in heavy nuclei because of the presence of intruder states of unatural parity in the main shells of given parity. However, as seen in FIG.1 first panel, the concentration of Cooper pairs also occurs in such light nuclei as the oxygen isotopes where no intruders are present. This means that pairing in nuclei is sufficiently strong so that κ grabs contributions from several main shells, allowing for parity mixing even in light nuclei. If one artificially restricted the pairing configurations, e.g. in ^{22}O, to the s-d shell, then certainly no concentration effect at all would be seen (in this respect, see also the study in [6]). So to grasp the full physics of nuclear pairing it is very important to work in a large configuration space, comprising several shells below and above the active one (see also [8]).

In order to understand in more detail where this extraordinary concentration effect from even-odd parity mixing comes from, let us consider a very simple model. We got inspired by the Thomas-Fermi model presented in Ref.[3] where the anomalous density matrix is given by $\kappa^{TF}(R,r) \sim k_F(R)j_0(k_F(R)r)$ with $j_0(x)$ a spherical Bessel fonction, $k_F(R) = \sqrt{\frac{2m}{\hbar^2}(\mu - V(R))}\ \Theta(\mu - V)$ the local Fermi momentum, μ the chemical potential (or Fermi energy), and $V(R)$ is phenomenological mean field potential. It can be shown [18] that a slightly more elaborate semiclassical version can be written as

$$\kappa^{sc}(R,r) = \frac{m}{\hbar^2\pi^2}\int dE\ \kappa(E)\ k_E(R)\ j_0(k_E(R)r)\ , \quad (5)$$

where $k_E(R)$ is the local momentum at energy E, obtainable from k_F in replacing μ by E, and $\kappa(E)$ is the continuum version of the κ's for the individual quantum levels: $\kappa(E) = \Delta(E)/(2\sqrt{(E-\mu)^2 + \Delta(E)^2})$. We see that for very small $\Delta's$, one gets back the TF model [3]. However, for realistic gap values the distribution of κ's is very important, otherwise the concentration effect will not show up. For $\Delta(E)$ we adjust a Fermi function to represent on average the gap- values of the individual single particle levels. An example can be seen on FIG.4 of Ref.[20]. In the present work, we have fitted the function $\Delta(E)$ on HFB-D1S results for ^{120}Sn. A good fit function is given by $\Delta(E) = 4/[1 + exp(E - \mu)/20]$ (all numbers are in MeV). For the mean field potential $V(R)$ we take the Woods-Saxon form of Ref. [17]. The chemical potential μ is determined, as usual, via the particle number condition.

In FIG.5 we show the corresponding semiclassical probability $P^{sc}(R,r)$. We see qualitatively good agreement with the quantal HFB-results, for instance in what concerns the concentration of small Cooper pairs in the sur-

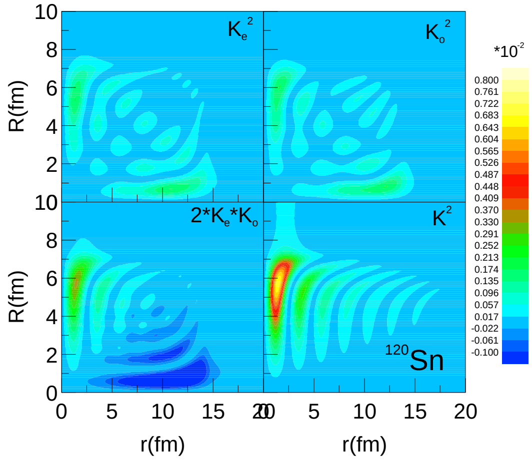

FIG. 5: Parities contributions $P^{sc}(R,r)$ calculated with the semiclassical model for ^{120}Sn.

face. We also show in FIG.5 the parity projected probabilities. As in FIG.4, one sees the strong delocalisation effect. In our model this can be understood analytically. Parity projection can be written as $\kappa_{e/o}(\vec{r}_1,\vec{r}_2) = \frac{1}{2}[\kappa(\vec{r}_1,\vec{r}_2) \pm \kappa(\vec{r}_1,-\vec{r}_2)] = \frac{1}{2}[\kappa(\vec{R},\vec{r}) \pm \kappa(\frac{\vec{r}}{2},2\vec{R})]$ (see Ref. [19]). We therefore see that good parity implies, up to a scale factor, a symmetrisation in coordinates R and r. This is general and can be investigated analytically in the TF model. We also calculate the coherence length semiclassicaly. We find qualitatively the same behavior as in the quantal calculation of FIG.3.
The analytic model also allows to quickly grasp the significance of the coordinates used by Matsuo et al. [8]. There, one takes a reference particle at position $\vec{r}_1$ on the z-axis, i.e. $\vec{r}_1 = z_1\vec{e}_z$. Moving the second particle on the z-axis we see that for $P_{e/o}$ two symmetric peaks at $\vec{r}_2 = z_1\vec{e}_z$ and $\vec{r}_2 = -z_1\vec{e}_z$ appear whereas for the total probability only one peak on the side of the test particle appears. This is a clear signature of strong pairing correlations as also pointed out in [8].

In conclusion, we showed that Cooper pairs in superfluid nuclei preferentially are located with small size $(2-3fm)$ in the surface region. There, they maximally profit from the Cooper phenomenon, that is, with respect to the neutron-neutron virtual S-state in the vacuum (rms 12 fm, [5]), strong extra binding occurs, as long as the density is not too high. Further to the center of the nucleus the stronger effect of the orthogonalisation of the pair with respect to the denser core-neutrons perturbs the pair wave function. It starts to oscillate and expands again [5]. That this simple, physically appealing and generic picture, is so pronounced, has come as a surprise. It is certainly important for the interpretation of pair transfer reactions. Most of these facts had already been revealed in the past for specific examples and schematic models and forces. We think, it is the merit of this work that it demonstrates with realistic HFB calculations using the finite range D1S force the generic aspect of strong coupling features of singlet isovector pairing in nuclei. These features are in agreement with the ones recently put forward by Matsuo et al. [8]. Let us mention that the strong coupling features revealed here are somewhat contrary to the old believe [22] that the coherence length of nuclear pairs is of the same order or larger than the nuclear diameter. On the contrary, a much more diverse local picture has emerged. This may also be the reason for the rather good succes of LDA for nuclear pairing found in the past [21]. In spite of the strong coupling aspects revealed in this work, we hesitate to say that there is Bose-Einstein condensation (BEC) of isovector Cooper pairs, since this, strictly speaking, occurs only for (in infinite matter) negative chemical potential, what means true binding. However, μ never turns negative for isovector pairing in infinite nuclear or neutron matters. Nuclear isovector pairing is just in the transition region from BEC to BCS.
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